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Charged lepton electric dipole moments with localized leptons and new Higgs doublet in the two Higgs doublet model

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Abstract. We study the electric dipole moments of the leptons in the split fermion scenario, in the two Higgs doublet model, where the new Higgs scalars are localized around the origin in the extra dimension, with the help of the localizer field. We observe that the numerical value of the electric dipole moment of the electron (muon, tau) is of the order of the magnitude of 10^{-31} (10^{-24}, 10^{-22}) ($e\,$cm), and this quantity is sensitive to the new Higgs localization in the extra dimension.

1 Introduction

The electric dipole moments (EDMs) of fermions are worthwhile to study, since they are driven by the CP-violating interactions. CP-violation is carried by the complex Cabibo–Kobayashi–Maskawa (CKM) matrix elements in the quark sector and the possible lepton mixing matrix elements in the lepton sector, in the framework of the standard model (SM). The negligibly small theoretical values of the EDMs of fermions in the SM forces one to search for physics beyond the SM, such as multi Higgs doublet models (MHDM), the supersymmetric model (SUSY) [1], etc.

There are various experimental results on the fermion EDMs in the literature, and they read $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}\, e\,\mathrm{cm}$ [2], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}\, e\,\mathrm{cm}$ [3] and $|d_\tau| < (3.1) \times 10^{-16}\, e\,\mathrm{cm}$ [4], respectively.

From the theoretical point of view, extensive work has been done on the EDMs of the fermions in various models. The electric dipole moments of the leptons have been studied in the seesaw model in [5]. The electron EDM has been predicted to be of the order of $10^{-32}\, e\,\mathrm{cm}$, in the 2HDM, including tree level flavor changing neutral currents (FCNC) and complex Yukawa couplings, in [6]. The work [7] is devoted to the fermionic EDM moments in the SM with the inclusion of non-commutative geometry. In [8] ([9]), the electric dipole moments of fermions in the two Higgs doublet model with the inclusion of non-universal extra dimensions (in the split fermion scenario) have been analyzed. On the other hand, the EDMs of the quarks have been estimated in several models [10–18, 20–28] and the EDMs of nuclei, the deuteron, the neutron and some atoms have been predicted extensively [29–40].[1]

In the present work, we study the lepton EDMs in the two Higgs doublet model, in which the flavor changing (FC) neutral current vertices on the tree level are permitted and the CP-violating interactions are carried by complex Yukawa couplings. In the calculations, we include the effects of a possible new dimension, in the split fermion scenario, where the hierarchy of fermion masses comes from the overlap of the Gaussian profiles of the fermions in the extra dimension. In addition to the strong localization of the leptons, we consider the case that the new Higgs scalars are also localized around the origin, in the extra dimension. These localizations are the result of the non-vanishing couplings between leptons (new Higgs scalars) and the so called localizer field, which is odd under Z_2 reflection in the extra dimension. The split fermion scenario has been studied in several works in the literature [41–64]. In [41, 42], the fermion mass hierarchy has been introduced by assuming that the fermions are located at different points in the extra dimensions, with exponentially small overlaps of their wavefunctions. The phenomenologically reliable locations of left and right handed components of fermions in the extra dimensions and their roles on the mechanism of Yukawa hierarchies have been studied in [44]. Reference [45] was devoted to the prediction of the constraint on the split fermions in the extra dimensions by considering leptonic W decays and lepton violating processes. The CP-violation in the quark sector in the split fermion model was studied in [46], and in [47] the new configuration of split fermion positions in a single extra dimension and the physics of the kaon, the neutron and of B/D mesons have been analyzed to find stringent bounds on the size of the compactification scale $1/R$. The shapes and overlaps of the fermion wavefunctions in the split fermion model have been studied in [48, 49] and the rare processes in the split fermion scenario have been analyzed in [50]. Reference [9] ([61, 62]) was related to the electric dipole moments of charged leptons (the radiative lepton flavor vi-

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[1] For a recent review, see [19].

olating (LFV) decays, the LFV $Z \to l_i l_j$ decays) in the split fermion scenario. Reference [63] was devoted to the branching ratios of the radiative LFV decays in the split fermion scenario, with the assumption that the new Higgs doublet is restricted to the 4D brane or to the thin bulk in one and two extra dimensions, in the framework of the two Higgs doublet model. Recently, the Higgs localization in the split fermion models has been studied in [64].

The present work is devoted to the EDMs of charged leptons in the 2HDM with the assumption that they have Gaussian profiles in the extra dimension, similar to the previous work [9]. However, we also consider the case that the new Higgs scalars are localized in the extra dimension with the help of the so called localizer field. The idea of the localization of the SM Higgs boson, using the localizer field, has been studied in [64]. Here, we assume that the new Higgs scalars are localized in the extra dimension and the SM Higgs boson has a constant profile. The localization of the new Higgs scalars depends strongly on the strength of the coupling of the localizer to the new Higgs scalar. Here we take this coupling small and study the sensitivity of the EDMs of the leptons to it. Furthermore, we analyze the compactification scale dependence of the lepton EDMs. In the numerical calculations, we observe that the lepton EDMs are sensitive to the new Higgs localization in the extra dimension, and we estimate that the numerical values of the electron (muon, tau) EDM is of the order of the magnitude of 10^{-31} (10^{-24}, 10^{-22}) $e\,$cm, even for large values of the compactification scale $1/R$ and for intermediate values of the Yukawa couplings.

The paper is organized as follows: in Sect. 2, we present the EDMs of charged leptons in the split fermion scenario, with the localization of new Higgs scalars in the extra dimension, in the 2HDM. Section 3 is devoted to a discussion and to our conclusions. In the appendix we present the derivation of the KK modes of new Higgs fields and their masses in the case that they are coupled to the localizer field.

2 The effect of the localization of new Higgs scalars on the electric dipole moments of charged leptons, in the split fermion scenario, in the two Higgs doublet model

The existence of the fermionic EDM is the signal for CP-violation, since it is the result of the CP-violating fermion–fermion–photon effective interaction. In the framework of the SM, the complex CKM matrix (possible lepton mixing matrix) is a possible source of this violation for quarks (for charged leptons). Since the theoretically estimated numerical values of the fermion EDMs are considerably small, one needs to go beyond the SM to get additional contributions, in order to enhance their numerical values. The multi Higgs doublet models are among the candidates for this enhancement, and, in the present work, we take the 2HDM, which allows for the FCNC at tree level and includes the complex Yukawa couplings as a source of CP-violation. On the other hand, with the addition of extra dimensions, there exist new contributions sensitive to the compactification scale $1/R$, where R is the compactification radius of the extra dimension. Here, we take the effects of the extra dimensions into account with the assumption that the hierarchy of lepton masses comes from the lepton Gaussian profiles in the extra dimensions, the so called the split fermion scenario. This localization is obtained by the nonzero coupling of the localizer scalar field ϕ_L to the lepton fields. Furthermore, we assume that this field couples also to the new Higgs doublet and localizes the new Higgs scalars around the origin $y = 0$, where y is the coordinate of the extra dimension (see the appendix).

The Yukawa Lagrangian that creates the lepton EDM in a single extra dimension, respecting the split fermion scenario, reads

$$\mathcal{L}_{\rm Y} = \xi^{E}_{5ij} \bar{\hat{l}}_{iL} \phi_2 \hat{E}_{jR} + \text{h.c.}\,, \tag{1}$$

where L and R denote the chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, and ϕ_2 is the new scalar doublet. Here $\hat{l}_{iL}$ ($\hat{E}_{jR}$), with family indices i, j, are the zero mode[2] lepton doublets (singlets) with Gaussian profiles in the extra dimension y; they read

$$\begin{aligned} \hat{l}_{iL} &= N \mathrm{e}^{-(y-y_{iL})^2/2\sigma^2} l_{iL}\,, \\ \hat{E}_{jR} &= N \mathrm{e}^{-(y-y_{jR})^2/2\sigma^2} E_{jR}\,, \end{aligned} \tag{2}$$

with the normalization factor $N = \frac{1}{\pi^{1/4}\sigma^{1/2}}$. l_{iL} (E_{jR}) are the lepton doublets (singlets) in four dimensions. Here the parameter σ is the Gaussian width of the leptons with the property $\sigma \ll R$, and the $y_{i(L,R)}$ are the fixed positions of the ith left (right) handed lepton in the fifth dimension.[3]

The Higgs doublets ϕ_1 and ϕ_2 are chosen as follows:

$$\begin{aligned} \phi_1 &= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ \mathrm{i}\chi^0 \end{pmatrix} \right]\,; \\ \phi_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + \mathrm{i}H_2 \end{pmatrix}\,, \end{aligned} \tag{4}$$

so that their vacuum expectation values read

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}\,; \quad \langle \phi_2 \rangle = 0\,. \tag{5}$$

This leads to the possibility of collecting SM (new) particles in the first (second) doublet. In this case H_1 and H_2

[2] In our calculations, we take only zero mode lepton fields.

[3] The positions of left handed and right handed leptons are obtained by taking the observed masses into account [44]. The idea is that the lepton mass hierarchy is due to the relative positions of the Gaussian peaks of the wavefunctions located in the extra dimension [41, 42, 44]. By assuming that the lepton mass matrix is diagonal, one possible set of locations for the lepton fields reads (see [44] for details)

$$P_{l_i} = \sqrt{2}\sigma \begin{pmatrix} 11.075 \\ 1.0 \\ 0.0 \end{pmatrix}\,, \quad P_{e_i} = \sqrt{2}\sigma \begin{pmatrix} 5.9475 \\ 4.9475 \\ -3.1498 \end{pmatrix}\,. \tag{3}$$

are the mass eigenstates h^0 and A^0, respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 at tree level.[4]

Now we take the case that the SM Higgs has a constant profile in the extra dimension, and the new Higgs field ϕ_2, which has the main role in the existence of the charged lepton EDM, couples to the localizer ϕ_L with a small coupling g (see the appendix for details). The coupling of the new Higgs doublet to the localizer results in the localization of the Higgs scalars, with the corresponding KK modes, around the origin, and brings about modified Yukawa interactions in four dimensions. The neutral CP-even and -odd scalar fields S ($S = h^0, A^0$) are expanded into their KK modes by $S(x,y) = \sum_n h_n(y) S^{(n)}(x)$ (see the appendix), and to obtain the lepton–lepton–Higgs interaction coupling in four dimensions we need to integrate the combination $\bar{\hat{f}}_{iL(R)} S^{(n)}(x) h_n(y) \hat{f}_{jR(L)}$, appearing in the part of the Lagrangian (1), over the fifth dimension. Using the KK basis obtained (see (A.6) and (A.8)), we get

$$\int_{-\pi R}^{\pi R} \mathrm{d}y \bar{\hat{f}}_{iL(R)} S^{(n)}(x) h_n(y) \hat{f}_{jR(L)} = V^n_{LR(RL)ij} \bar{f}_{iL(R)} \times S^{(n)}(x) f_{jR(L)} \,, \tag{7}$$

where the factor $V^n_{LR(RL)ij}$ reads

$$V^n_{LR(RL)ij} = V^0_{LR(RL)ij} c^{(\prime)}_n(i,j) \,, \tag{8}$$

and the function $V^0_{LR(RL)ij}$ is

$$V^0_{LR(RL)ij} = \frac{g^{1/8} \mathrm{e}^{-(y_{iL(R)} - y_{jR(L)})^2/4\sigma^2}}{\sqrt{\sigma}\pi^{1/4}\sqrt{\mathrm{Erf}\left[\pi g^{1/4}\frac{R}{\sigma}\right]}} \,. \tag{9}$$

Here, the fields f_{iL} and f_{jR} are the four dimensional lepton fields, and the function Erf$[z]$ is the error function, which is defined as

$$\mathrm{Erf}[z] = \frac{2}{\sqrt{\pi}} \int_0^z \mathrm{e}^{-t^2} \,\mathrm{d}t \,. \tag{10}$$

[4] Here we consider the Higgs potential term $V(\phi_1, \phi_2, \phi_L)$

$$V(\phi_1,\phi_2,\phi_L) = c_1\left(\phi_1^+\phi_1 - v^2/2\right)^2 + c_2\left(\phi_2^+\phi_2\right)^2 + \frac{1}{2} g \phi_2^\dagger \phi_2 \phi_L^2 \,, \tag{6}$$

with constants $c_i, i = 1,2$, where ϕ_L is the localizer singlet field (see the appendix for details). In this case, the first Higgs doublet does not couple to the second one and to the localizer singlet field. With this assumption, the two Higgs doublets do not mix and the SM (new) particles are placed in the first (second) doublet. If the first and the second doublets couple, and the second one acquires a non-zero vacuum expectation value, mixing appears between the two doublets; this needs further analysis.

The functions $c^{(\prime)}_n(i,j)$ in (8) are calculated in the case that the coupling g is non-zero and small:

$$c^{(\prime)}_n(i,j) = \frac{(-1)^n 2^{3n+\frac{1}{2}} \sqrt{\pi}\left(1-\frac{g}{4}\right)^n \left(2+\sqrt{g}\right)^{-\frac{4n+1}{2}}}{\sqrt{(2n)!}\Gamma\left[\frac{1}{2}-n\right]} \times \mathrm{e}^{f_{LR(RL)ij}} {}_1F_1\left[-n, \frac{1}{2}, \frac{\sqrt{g}(y_{iL(R)}+y_{jR(L)})^2}{(4-g)\sigma^2}\right] , \tag{11}$$

where

$$f_{LR(RL)ij} = -\frac{\sqrt{g}(y_{iL(R)}+y_{jR(L)})^2}{4(2+\sqrt{g})\sigma^2} \,. \tag{12}$$

The function ${}_1F_1[a;b;z]$ appearing in (11) is the hypergeometric function

$${}_1F_1[a;b;z] = \sum_{k=0}^{\infty} (a)_k (b)_k z^k / k! \,, \tag{13}$$

where $(d)_k = \frac{\Gamma[d+k]}{\Gamma[d]}$. We have

$$\xi^E_{ij} \left(\left(\xi^E_{ij}\right)^\dagger\right) = V^0_{LR(RL)ij} \xi^E_{5ij} \left(\left(\xi^E_{5ij}\right)^\dagger\right) , \tag{14}$$

where ξ^E_{5ij} are the Yukawa couplings in five dimensions (see (1)).[5]

The effective EDM interaction for a charged lepton f is given by

$$\mathcal{L}_{\mathrm{EDM}} = \mathrm{i} d_f \bar{f} \gamma_5 \sigma^{\mu\nu} f F_{\mu\nu} \,, \tag{15}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, 'd_f' is the EDM of the charged lepton, and it is a real number by hermiticity. With the assumption that the CP-violating EDM interaction comes from the complexity of the Yukawa couplings due to the new Higgs scalars,[6] the f-lepton EDM 'd_f' ($f = e, \mu, \tau$) can be calculated as a sum of contributions coming from the neutral Higgs bosons h_0 and A_0 (see Fig. 1),

$$d_f = -\frac{\mathrm{i}G_{\mathrm{F}}}{\sqrt{2}} \frac{e}{32\pi^2} \frac{Q_\tau}{m_\tau} \left(\left(\bar{\xi}^{D*}_{\mathrm{N},l\tau}\right)^2 - \left(\bar{\xi}^{D}_{\mathrm{N},\tau l}\right)^2\right) \Bigg(c_0(f,\tau) c'_0(f,\tau) \times \left(F_1(y_{h_0}) - F_1(y_{A_0})\right) + \sum_{n=1}^{\infty} c_n(f,\tau) c'_n(f,\tau) \Big(F_1\left(y^n_{h_0}\right) - F_1\left(y^n_{A_0}\right) \Big) \Bigg) , \tag{16}$$

[5] In the following we use the dimensionful coupling $\bar{\xi}^E_{\mathrm{N}}$ in four dimensions, with the definition $\xi^E_{\mathrm{N},ij} = \sqrt{\frac{4G_{\mathrm{F}}}{\sqrt{2}}} \bar{\xi}^E_{\mathrm{N},ij}$ where N denotes the word "neutral".

[6] We do not consider the possible effects due to the CKM type lepton mixing matrix and take only zero mode lepton fields.

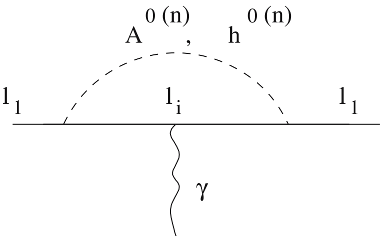

Fig. 1. One loop diagrams contribute to EDM of charged leptons due to neutral Higgs bosons h^0, A^0 in the 2HDM, including KK modes in a single extra dimension. *Wavy lines* represent the electromagnetic field and *dashed lines* the Higgs field where $l_{1(i)} = e, \mu, \tau$

for $f = e, \mu$, and

$$\begin{aligned} d_\tau = & -\frac{\mathrm{i}G_{\mathrm{F}}}{\sqrt{2}}\frac{e}{32\pi^2}\Bigg\{\frac{Q_\tau}{m_\tau}\left(\left(\bar{\xi}^{E*}_{\mathrm{N},\tau\tau}\right)^2-\left(\bar{\xi}^{E}_{\mathrm{N},\tau\tau}\right)^2\right) \\ & \times\Bigg(c_0^2(\tau,\tau)\left(F_2(r_{h_0})-F_2(r_{A_0})\right) \\ & +\sum_{n=1}^{\infty}c_n^2(\tau,\tau)\left(F_2\left(r^n_{h_0}\right)-F_2\left(r^n_{A_0}\right)\right)\Bigg) \\ & -Q_\mu\frac{m_\mu}{m_\tau^2}\left(\left(\bar{\xi}^{E*}_{\mathrm{N},\mu\tau}\right)^2-\left(\bar{\xi}^{E}_{\mathrm{N},\tau\mu}\right)^2\right) \\ & \times\Bigg(c_0(\mu,\tau)c_0'(\mu,\tau)(r_{h_0}\ln(z_{h_0})-r_{A_0}\ln(z_{A_0})) \\ & +\sum_{n=1}^{\infty}c_n(\mu,\tau)c_n'(\mu,\tau)\Big(r^n_{h_0}\ln\left(z^n_{h_0}\right) \\ & -r^n_{A_0}\ln\left(z^n_{A_0}\right)\Big)\Bigg)\Bigg\}, \end{aligned} \tag{17}$$

for $f = e, \mu, \tau$. Here the functions $F_1(w)$ and $F_2(w)$ read

$$\begin{aligned} F_1(w) &= \frac{w(3-4w+w^2+2\ln w)}{(-1+w)^3}, \\ F_2(w) &= w\ln w+\frac{2(-2+w)w\ln\frac{1}{2}(\sqrt{w}-\sqrt{w-4})}{\sqrt{w(w-4)}}, \end{aligned} \tag{18}$$

with $y_S = \frac{m_\tau^2}{m_S^2+2\beta}$, $y_S^n = \frac{m_\tau^2}{m_S^2+2(4n+1)\beta}$, $r_S = \frac{1}{y_S}$, $r_S^n = 1/y_S^n$, $z_S = \frac{m_\mu^2}{m_S^2+2\beta}$, $z_S^n = \frac{m_\mu^2}{m_S^2+2(4n+1)\beta}$, and the Q_τ and Q_μ are the charges of the τ and μ leptons, respectively. Notice that the functions $c_n^{(\prime)}(i,j)$[7] are defined in (11). In (16) we take into account only the internal τ-lepton contribution respecting our assumption that the Yukawa couplings $\bar{\xi}^E_{\mathrm{N},ij}$, $i,j = e, \mu$, are small compared to $\bar{\xi}^E_{\mathrm{N},\tau i}$, $i = e, \mu, \tau$, due to the possible proportionality of the Yukawa couplings to the masses of leptons in the vertices. In (17), we also present the internal μ-lepton contribution, which can be neglected numerically. Notice that we make our calculations for arbitrary photon four momentum squared q^2 and take $q^2 = 0$ at the end.

Finally, in our calculations, we choose the Yukawa couplings complex, and we use the parametrization

$$\bar{\xi}^E_{\mathrm{N},\tau f} = |\bar{\xi}^E_{\mathrm{N},\tau f}|\mathrm{e}^{\mathrm{i}\theta_f}. \tag{19}$$

Therefore, the Yukawa factors in (16) and (17) can be written as

$$\left(\left(\bar{\xi}^{E*}_{\mathrm{N},f\tau}\right)^2-\left(\bar{\xi}^{E}_{\mathrm{N},\tau f}\right)^2\right) = -2\mathrm{i}\sin 2\theta_f|\bar{\xi}^E_{\mathrm{N},\tau f}|^2, \tag{20}$$

where $f = e, \mu, \tau$. Here θ_f is the CP-violating parameter, which is the source of the EDM of the lepton.

3 Discussion

The fermionic EDM is the result of the existence of the CP-violation and arises from the fermion–fermion–photon effective interaction. In the present work, we study the charged lepton EDMs in the 2HDM with FCNC at tree level, and we assume that the CP phase comes from the complex Yukawa couplings, driving the lepton–lepton–new Higgs scalar vertices. Furthermore, we take into account the effects of a single extra dimension in the so called split fermion scenario, where the hierarchy of the lepton masses comes from the lepton Gaussian profiles in the extra dimension. In this scenario, the penetration of leptons into the bulk is ensured by the non-zero coupling of leptons with a background scalar field, namely the localizer field, ϕ_L, which has a vacuum expectation value, depending on the coordinate of the extra dimension, and centered at the origin. In addition to this, we consider the case that the new Higgs doublet also weakly couples to the localizer and the corresponding Higgs scalars localize around the origin $y = 0$. On the other hand, the SM Higgs H^0 is taken as uniform in the extra dimension, and its vacuum expectation value is responsible for the generation of the masses. Notice that the mass term, which is modulated by the mutual overlap of the lepton wavefunctions, is obtained by integrating the operator $H^0\bar{\hat{f}}\hat{f}$ over the extra dimensions, where f denotes a lepton. This is the idea of fixing the positions of left (right) handed leptons in the extra dimensions (see [44] for details). Since the lepton–lepton–new scalar vertices create the CP-violating EDM interaction, besides the zero modes, the KK modes of leptons and Higgs scalars have additional contributions. Here, we include the effects of the Higgs KK modes; however, we ignored the lepton KK mode contributions to the EDMs of the leptons. We expected that, for large values of the compactification scale, their effects on the physical parameters are suppressed.

In the numerical calculations, the free parameters occurring in the model used should be fixed by using the present experimental results. The first set of free param-

[7] If the second Higgs doublet is localized around $y = y_0$, the zero mode solution reads $h_0(y) = N'\mathrm{e}^{-\beta(y-y_0)^2}$ (see (A.6)), and, if y_0 is away from the origin, $y_0 \to \pi R$, the coefficients $c_n^{(\prime)}(i,j)$ tend to zero. This is the case in which the new Higgs contribution vanishes.

eters is the four dimensional leptonic complex couplings $\bar{\xi}^E_{N,ij}$, $i,j=e,\mu,\tau$. We consider the Yukawa couplings $\bar{\xi}^E_{N,ij}$, $i,j=e,\mu$, to be smaller than $\bar{\xi}^E_{N,\tau i}$, $i=e,\mu,\tau$, and we assume that $\bar{\xi}^E_{N,ij}$ is symmetric with respect to the indices i and j. In the case that no extra dimension exists, the experimental uncertainty, 10^{-9}, in the measurement of the anomalous magnetic moment of the muon [65] can be used to estimate the upper limit of $\bar{\xi}^E_{N,\tau\mu}$, since the new physics effects cannot exceed this uncertainty, and this limit was predicted to be 30 GeV (see [66] and references therein). Using this upper limit and the experimental upper bound of the BR of the $\mu\to e\gamma$ decay, $\mathrm{BR}\le 1.2\times 10^{-11}$, the coupling $\bar{\xi}^E_{N,\tau e}$ is restricted to the range 10^{-3}–10^{-2} GeV [6]. In our calculations, we choose the numerical values of the couplings $\bar{\xi}^E_{N,\tau\mu}$ ($\bar{\xi}^E_{N,\tau e}$) to be no larger than 30 GeV (around 10^{-3} GeV). For the coupling $\bar{\xi}^E_{N,\tau\tau}$, we use numerical values that are greater than $\bar{\xi}^E_{N,\tau\mu}$, since we have no explicit restriction region. For the CP-violating parameter, which drives the EDM interaction, we choose the intermediate value $\sin\theta_{e(\mu,\tau)}=0.5$.

The second set of free parameters comes from the split fermion scenario, in which the hierarchy of the lepton masses is due to the Gaussian profiles of the leptons in the extra dimension. Here, we have a free parameter $\rho=\sigma/R$, where σ is the Gaussian width of the fermions and R is the compactification radius. There are various predictions on the compactification radius in the literature. The lower bound for the inverse of the compactification radius was estimated as ~ 300 GeV [67, 68] in the case of the universal extra dimension scenario. The direct limits from searching for KK gauge bosons imply $1/R>800$ GeV, the precision electroweak bounds on higher dimensional operators generated by KK exchange place the far more stringent limit $1/R>3.0$ TeV [69], and, from $B\to\phi K_S$, the lower bounds for the scale $1/R$ have been obtained to be $1/R>1.0$ TeV; from $B\to\psi K_S$ one got $1/R>500$ GeV, and from the upper limit of the BR, $\mathrm{BR}(B_s\to\mu^+\mu^-)<2.6\times 10^{-6}$, the estimated limit was $1/R>800$ GeV [50]. On the other hand, there exists a different phenomenologically reliable set of locations of lepton fields in the extra dimension, even if they are predicted by using the lepton masses. Notice that we used the set[8] predicted by [44] . Finally, the coupling g, which exists since we consider the case that the new Higgs doublet also couples to the localizer, is another free parameter of the model used. Here, we assume that this coupling is in a region such that the new Higgs scalars are localized around the origin as much as possible. Notice that the masses of the zero mode Higgs boson h^0 and A^0, $m^2_{0,h^0(A^0)}=m^2_{h^0(A^0)}+2\beta$ (with $\beta=\sqrt{g}\mu^2$), depend on the coupling g and the additional term comes from the localization effect, which should not be large.[9]

Now, we start an estimate of the charged lepton EDMs and study the coupling g and the dependence of these physical quantities on the parameter ρ.

In Fig. 2, we plot the electron EDM d_e with respect to the coupling g for $1/R=1000$ GeV and $\bar{\xi}^E_{N,\tau e}=0.001$ GeV. Here the lower–upper solid (dashed) line represents the EDM, due to the zero mode Higgs–KK mode Higgs included contribution for the parameter $\rho=0.001$ ($\rho=0.0005$). The sensitivity of EDM to the coupling g is strong. The addition of KK mode contributions enhances the EDM weakly, especially for the parameter value $\rho=0.001$. The EDM of the electron is at most of the order of the magnitude of 10^{-31}–10^{-32} (e cm), for $g\sim 10^{-14}$ and $\rho=0.001$ ($\rho=0.0005$). The enhancement in EDM for the small values of the coupling g (and also for the large values of the width σ) is due to the fact that the masses of the zero (and also KK) modes (see (A.9)) are smaller than the ones for the large values of the coupling g (and also for small values of the width σ). In the case that all the 2HDM particles are confined to the 4D brane and the extra dimension effects are switched off, the EDM of the electron d_e is of the order of 10^{-30} (e cm), which is the numerical value to be reached with smaller coupling g and weak localization of the new Higgs doublet, in the present scenario.

Figure 3 is devoted to the dependence of d_e on the compactification scale $1/R$, for $g=10^{-13}$ and $\bar{\xi}^E_{N,\tau e}=0.001$ GeV. Here the lower–upper solid (dashed) line represents the d_e, due to the zero mode Higgs–KK mode Higgs included contribution for the parameter $\rho=0.001$ ($\rho=0.0005$). The sensitivity of d_e to $1/R$ is strong, and it enhances with decreasing values of $1/R$, as expected. The effects of the addition of the KK modes on d_e are weak.

In Fig. 4, we present d_μ with respect to the coupling g for $1/R=1000$ GeV, $\bar{\xi}^E_{N,\tau\mu}=10$ GeV. Here the lower–upper solid (dashed) line represents the EDM, due to the zero mode Higgs–KK mode Higgs included contribution for the parameter $\rho=0.001$ ($\rho=0.0005$). Similar to the EDM of the electron, the sensitivity of d_μ to the coupling

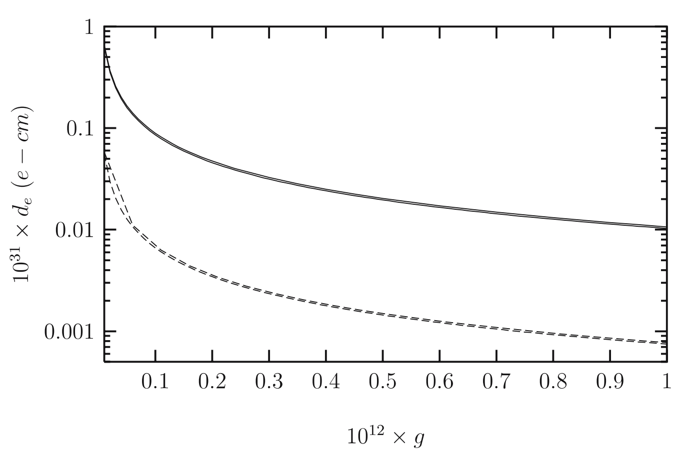

Fig. 2. d_e with respect to the coupling g for $1/R=1000$ GeV, $\bar{\xi}^E_{N,\tau e}=0.001$ GeV. Here the *lower–upper solid* (*dashed*) *line* represents the EDM, due to the zero mode Higgs-KK mode Higgs included contribution for the parameter $\rho=0.001$ ($\rho=0.0005$)

[8] The numerical results of the physical parameters depend on the choice of the set of locations of the lepton fields.

[9] Here we choose the masses coming from the Higgs mass term m_S in the Lagrangian (see (A.2)) as $m_{h^0}\sim 0$ (GeV) and $m_{A^0}\sim 180$ (GeV) and choose the coupling g in the range such that the zero mode masses $m_{0,h^0(A^0)}$ are around the numerical values 100 (200) GeV.

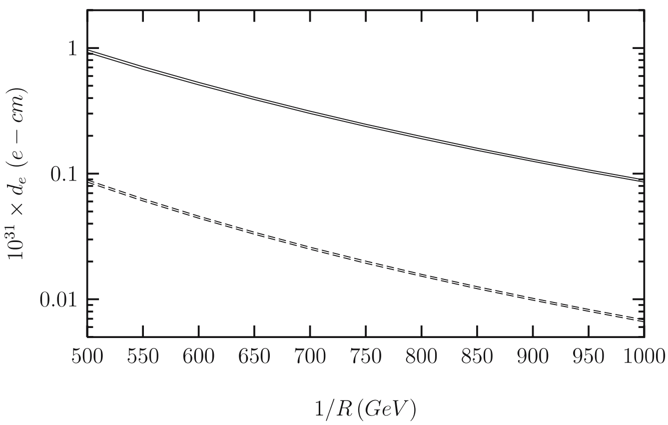

Fig. 3. d_e with respect to $1/R$, for $g = 10^{-13}$ and $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.001$ GeV. Here the *lower–upper solid* (*dashed*) *line* represents the d_e, due to the zero mode Higgs-KK mode Higgs included contribution for the parameter $\rho = 0.001$ ($\rho = 0.0005$)

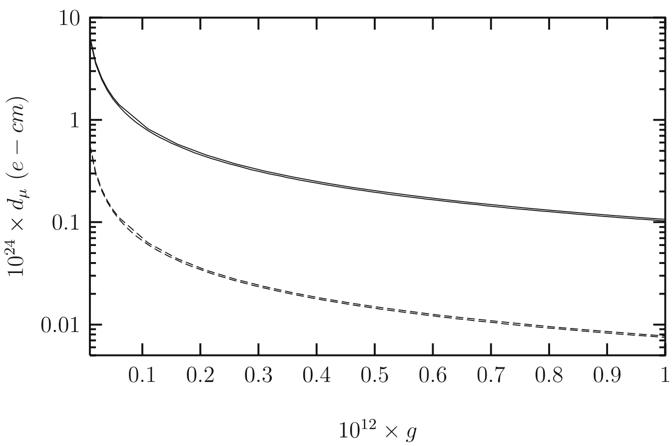

Fig. 4. The same as Fig. 2, but for d_μ and $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10$ GeV

g is strong, and the addition of the KK modes of the new Higgs scalars ensures a negligible enhancement in the numerical value of d_μ. The EDM of the muon is of the order of 10^{-23}–10^{-24} (e cm), for $g \sim 10^{-14}$ and $\rho = 0.001$ ($\rho = 0.0005$). The $1/R$ dependence of d_μ is presented in Fig. 5, for $g = 10^{-13}$ and $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.001$ GeV. Here the lower–upper solid (dashed) line represents the d_μ, due to the zero mode Higgs–KK mode Higgs included contribution for the parameter $\rho = 0.001$ ($\rho = 0.0005$). The sensitivity of d_μ to $1/R$ is strongly similar to the d_e case. If the extra dimension effects are switched off, the EDM of the muon d_μ is of the order of 10^{-22} (e cm), and this numerical value can be obtained with the weak localization of the new Higgs doublet, in the present scenario.

Finally, we study the EDM of the tau, d_τ, in Figs. 6 and 7. Figure 6 represents d_τ with respect to the coupling g for $1/R = 1000$ GeV, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10$ GeV and $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 50$ GeV. Here the lower–upper solid line represents the EDM, due to the zero mode Higgs–KK mode Higgs included contribution for the parameter $\rho = 0.001$. We observe that d_τ is enhanced up to values of the order of 2×10^{-22} (e cm)

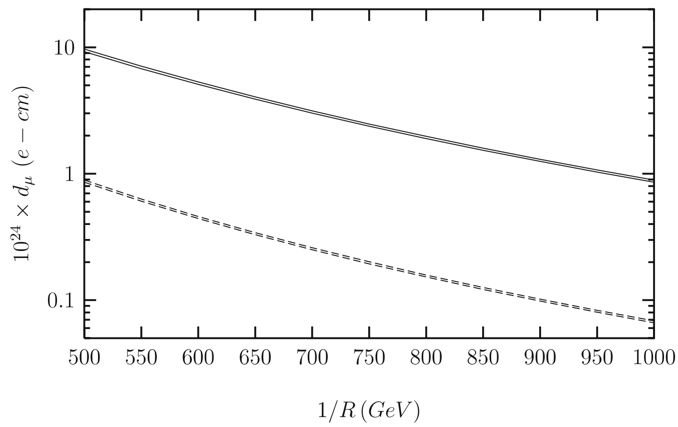

Fig. 5. The same as Fig. 3, but for d_μ and $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10$ GeV

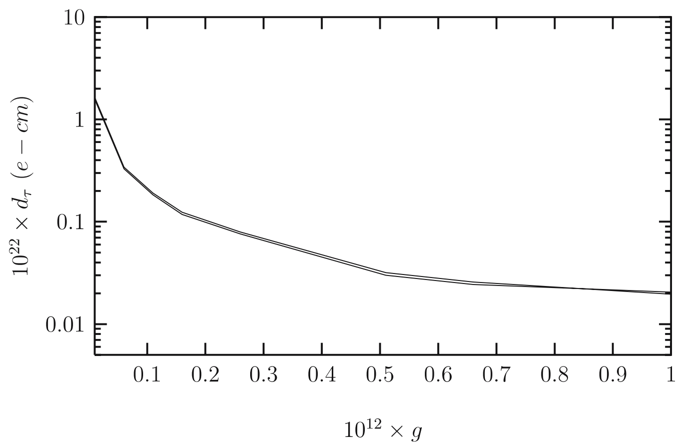

Fig. 6. The same as Fig. 2, but for d_τ and $\rho = 0.001$, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10$ GeV and $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 50$ GeV

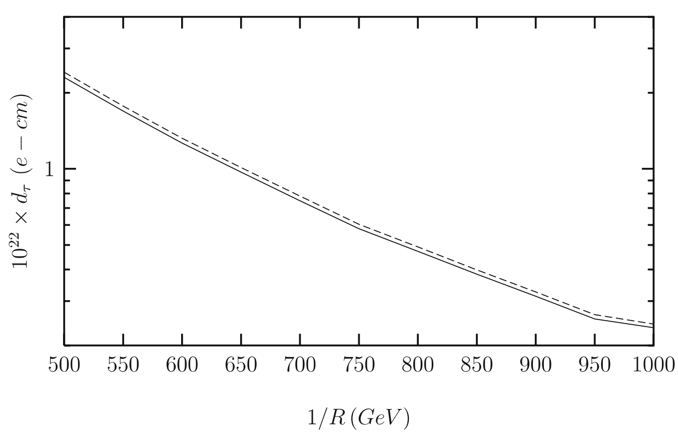

Fig. 7. The same as Fig. 3, but for d_τ and $\rho = 0.001$, $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10$ GeV and $\bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 50$ GeV

for $g \sim 10^{-14}$ and $\rho = 0.001$. The enhancement due to the KK modes of the new Higgs scalars is negligible. Figure 7 is devoted to the $1/R$ dependence of d_τ for $g = 10^{-13}$ and $\bar{\xi}^{E}_{\mathrm{N},\tau e} = 0.001$ GeV. Here the solid (dashed) line represents the d_τ, due to the zero mode Higgs (KK mode Higgs included) contribution for the parameter $\rho = 0.001$. The sensitivity of d_τ to $1/R$ is strong and shows an enhancement of almost one order with decreasing values of $1/R$ from 1000 (GeV) to 500 (GeV). The EDM of the tau, d_τ, is of the order of 10^{-21} (e cm) in the case that the extra dimension effects are switched off. Such a numerical value is obtained for the weak localization of the new Higgs doublet, similar to the EDM in case of the electron and muon.

Now we would like to summarize our results.

- The lepton EDMs are weakly sensitive to the KK mode contributions in the given range of the coupling g; however, this sensitivity increases for small values of this coupling. Furthermore, the numerical values of the EDMs increase with decreasing values of the coupling g and increasing values of the parameter ρ, which measures the Gaussian widths of the leptons in the extra dimension.
- The sensitivity of EDMs of the leptons to $1/R$ increases with decreasing values of the parameter ρ.
- We obtain the numerical values of the order of $\sim 10^{-31}$ (10^{-24}, E-21) (e cm) for $g \sim 10^{-14}$ and $\rho = 0.001$, for the EDMs d_e (d_μ, d_τ), respectively. To be able to reach the results near the current experimental limits, the following domains for the numerical values of the various free parameters should be considered: $\bar{\xi}^E_{\mathrm{N},\tau e} > 0.001$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau\mu} > 10$ GeV, $\bar{\xi}^E_{\mathrm{N},\tau\tau} > 50$ GeV, $1/R < 500$ GeV, $\rho > 0.001$ and $g \sim 10^{-14}$. It is obvious that these theoretical values of the EDMs of the charged leptons are still far from the experimental limits. Furthermore, the bound for the compactification scale almost coincides with the following bounds existing in the literature: $1/R \sim 300$ GeV [67, 68] in the case of the universal extra dimension scenario, and $1/R > 500$ GeV from $B \to \psi K_S$ [50]. Hopefully, the forthcoming more sensitive experimental measurements will ensure smaller numerical values for the EDMs of the leptons and, therefore, the possible range of the free parameter set will be more accurate.

As a final word, the future experimental measurements on the EDMs of the leptons would ensure valuable information on the possibility of the existence of localizations of leptons and Higgs bosons in the extra dimensions and make it clearer how nature behaves beyond the SM.

Appendix: The KK modes of the Higgs fields and their masses

Here, we calculate the KK modes of the Higgs fields and their masses in the case that the localizer field ϕ_L weakly couples to the new Higgs fields and localize these new fields around $y = 0$, where y is the coordinate of the extra dimension. Our starting point is the Lagrangian of the new scalar field, which is coupled to the localizer field:

$$L = \sum_{S=h^0,A^0} \left(\frac{1}{2} \partial_M S(x,y) \partial^M S(x,y) - V(S,\phi_L) \right) , \tag{A.1}$$

with $M = 0, 1, \ldots, 4$, and

$$\partial_M S(x,y) \partial^M S(x,y) = \partial_\mu S(x,y) \partial^\mu S(x,y) - \left(\frac{\mathrm{d}S(x,y)}{\mathrm{d}y} \right)^2 ,$$

with $\mu = 0, 1, 2, 3$. Here we consider the potential $V(S, \phi_L)$,

$$V(S,\phi_L) = \frac{1}{2} m_S^2 S^2(x,y) + \frac{1}{2} g S^2(x,y) \phi_L^2 , \tag{A.2}$$

where ϕ_L is the localizer singlet field, which is chosen as $\phi_L = 2\mu^2 y$ [41, 42]. It has a linear dependence on the new coordinate y for the points near the fixed point, and it couples weakly to the new scalar fields $S(x, y)$.[10] Now the Lagrangian L can be rewritten as

$$L = \frac{1}{2} \partial_\mu S(x,y) \partial^\mu S(x,y) - \frac{1}{2} S(x,y) \left(-\frac{\mathrm{d}}{\mathrm{d}y^2} + \frac{\partial^2 V}{\partial S^2} \right) \times S(x,y) . \tag{A.3}$$

With the following expansion over the KK modes

$$S(x,y) = \sum_r h_r(y) S^{(r)}(x) , \tag{A.4}$$

one gets the mass equation

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}y^2} + m_S^2 + g(2\mu^2)^2 y^2 \right) h_r(y) = m_r^2 h_r(y) . \tag{A.5}$$

The ground state (the zero mode) solution reads

$$h_0(y) = N \mathrm{e}^{-\beta y^2} , \tag{A.6}$$

with the normalization constant $N = \frac{(2\beta)^{1/4}}{\sqrt{\pi^{1/2} \mathrm{Erf}[\pi R \sqrt{2\beta}]}}$, and its corresponding mass $m_{0,S}$ obeys

$$m_{0,S}^2 = m_S^2 + 2\beta , \tag{A.7}$$

with $\beta = \sqrt{g}\mu^2$. Here μ is the energy scale that adjusts the localization of leptons in the extra dimension, namely, $\mu = \frac{1}{\sqrt{2}\sigma}$.

The KK modes for $n \neq 0$ can be obtained by using the operator $D = \frac{\mathrm{d}}{\mathrm{d}y'} - y'$ as follows:

$$h_r(y) = N^{(r)} D^r \mathrm{e}^{-\frac{1}{2} y'^2} , \tag{A.8}$$

where $y' = \sqrt{2\beta} y$ and $N^{(r)}$ is the normalization constant for the r_{th} KK mode, and $N^{(r)} = \frac{\beta^{1/4}}{\sqrt{\pi^{1/2} 2^{n-\frac{1}{2}} (n)!} \sqrt{\mathrm{Erf}[\pi R \sqrt{2\beta}]}}$.

Finally, the masses of the KK modes, including the zero one, become[11] and

$$m_{r,S}^2 = m_S^2 + 2\beta(2r+1) . \tag{A.9}$$

[10] We consider an additional interaction term $\frac{1}{2} g \phi_2^\dagger \phi_2 \phi_L^2$ in the full Lagrangian.

[11] Notice that we take the even KK modes of the new Higgs bosons, namely the ones with $r = 2n$, $n = 0, 1, \ldots$, since the extra dimension is compactified on to the orbifold S^1/Z_2. In this case, the KK mode masses read $m_{r,S}^2 = m_S^2 + 2\beta(4n+1)$

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